Uncertainty Quantification of Existing Bridge using Polynomial Chaos Expansion

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Abstract. *This paper is focused on uncertainty quantification (UQ) of an existing bridge structure represented by non-linear finite element model (NLFEM). The 3D model was created according to the original drawings and recent inspections of the bridge. In order to reflect the realistic mechanical behavior, the mathematical model is based on non-linear fracture mechanics and the calculation consists of the three construction stages. The single calculation of the NLFEM is very costly and thus even the elementary task of stochastic analysis – the propagation of uncertainties through a mathematical model – is not feasible by Monte Carlo-type approach. Thus, UQ is performed via efficient surrogate modeling technique – Polynomial Chaos Expansion (PCE). PCE is a well-known technique for approximation of the costly mathematical models with random inputs, reflecting their distributions and offering fast and accurate post-processing including statistical and sensitivity analysis. Once the PCE was constructed, it was possible to analyze all quantities of interest (QoIs) and analytically estimate Sobol indices as well as the first four statistical moments. Sobol indices directly measure the influence of the input variability to a variability of QoIs. Statistical moments were used for reconstruction of the probability distributions of QoIs, which will be further used for semi-probabilistic assessment. Moreover, once the PCE is available it could be possible to use it for further standard probabilistic or reliability analysis as a computationally efficient approximation of the original mathematical model.*

Keywords

Uncertainty quantification, polynomial chaos expansion, statistical analysis.

1. Introduction

The analysis of existing structures is of great importance since it is often necessary to assess the reliability of structures reaching their designed lifetime or adapt them to different loading conditions. Nowadays, it is common to create costly mathematical models as digital twins, which are able to greatly estimate real behavior of the existing structures. However, existing structures are associated with many uncertainties regarding material properties possibly affected by deterioration, quality of execution and maintenance, local environmental effects resulting in defects. Such uncertainties could play a crucial role in reliability assessment of existing structures [1] and thus it is necessary to extend deterministic analysis to stochastic analysis. The advanced stochastic analysis is especially valuable in the case of existing concrete structures such as bridges since there is a high variability of the basic variables assumed in the mathematical models [2]. Standard method for propagation of uncertainties through the mathematical model and analysis of QoI is Monte Carlo approach, which needs high number of repetitive simulations with randomly generated realizations of the input random vector.

The stochastic analysis of costly mathematical models often solved by finite element method is typically not feasible due to computational requirements caused by a combination of two aspects: large number of repetitive deterministic simulations used in probabilistic analysis with high computational cost per simulation. Therefore, it is often necessary to construct a computationally efficient approximation – surrogate model. Surrogate models are constructed from given experimental design, i.e. set of realizations of input random vector and corresponding results of the original mathematical model. Although there are various surrogate models in scientific literature, here we use PCE thanks to its possibilities for efficient and accurate UQ including statistical and sensitivity analysis. Analytical post-processing is a unique feature of PCE and it represents a significant advantage in comparison to other popular surrogate models such artificial neural networks [3, 4]. The paper is particularly focused on UQ of an existing concrete bridge using PCE constructed by advanced algorithms implemented in a software package UQPy.

2. Polynomial Chaos Expansion

2.1. Theoretical Background

Evaluation of a mathematical model representing physical system in civil engineering is typically costly and thus it is necessary to create an efficient approximation. One of the most popular approaches is PCE [5], which represents the output variable Y as a polynomial expansion g^{PCE} of an another random variable ξ called a germ with given distribution

$$Y = g(X) \approx g^{PCE}(\xi), \quad (1)$$

A set of polynomials, orthogonal with respect to the probability distribution of the germ, are used as a basis functions. The orthogonality condition for all $j \neq k$ is given by the inner product defined for any two functions ψ_j and ψ_k with respect to the probability density function of ξ

$$\langle \psi_j, \psi_k \rangle = \int \psi_j(\xi)\psi_k(\xi)p_\xi(\xi)\,\mathrm{d}\xi = 0. \quad (2)$$

Polynomials ψ orthogonal with respect to a selected probability distributions p_ξ can be chosen according to Wiener-Askey scheme [6] or created directly by Gram-Schmidt orthogonalization. In this paper we use normalized polynomials with inner product equal to the Kronecker delta δ_{jk}.

In the case of $\boldsymbol{X}$ and $\boldsymbol{\xi}$ being vectors containing M random variables, the polynomial $\Psi(\boldsymbol{\xi})$ is multivariate and it is built up as a tensor product of univariate orthogonal polynomials. The quantity of interest (QoI), i.e. the response of the mathematical model $Y = g(\boldsymbol{X})$, can then be represented, according to Ghanem and Spanos [7], as

$$Y = g(\boldsymbol{X}) = \sum_{\boldsymbol{\alpha}\in\mathbb{N}^M} \beta_{\boldsymbol{\alpha}}\Psi_{\boldsymbol{\alpha}}(\boldsymbol{\xi}), \quad (3)$$

where $\boldsymbol{\alpha} \in \mathbb{N}^M$ is a set of integers called the *multi-index*, $\beta_{\boldsymbol{\alpha}}$ are deterministic coefficients and $\Psi_{\boldsymbol{\alpha}}$ are multivariate orthogonal polynomials.

Naturally, the approximating function given by Eq. (3) must be truncated to a finite number of terms P. There are various schemes for truncation of multivariate basis sets (see Fig.1 for a comparison). The simplest approach is a common tensor product, though it suffers significantly from a *curse-of-dimensionality*. Therefore, common approach is total-order truncation by retaining only terms whose total degree $|\boldsymbol{\alpha}|$ is less than or equal to a given p:

$$\mathcal{A}^{M,p} = \left\{ \boldsymbol{\alpha} \in \mathbb{N}^M : |\boldsymbol{\alpha}| = \sum_{i=1}^{M} \alpha_i \leq p \right\}. \quad (4)$$

In case of high p and M, it possible to use additional “hyperbolic” reduction of the truncated set [8]. Hyperbolic truncation neglects high-order interaction terms, though it is described by hyper-parameter q which must be chosen in advance. However, if there is reliable information about the mathematical model (such as an effect of sparsity often present in physics phenomena), it is possible to drastically reduce number of basis functions and thus number of unknown deterministic coefficients. The most advanced but also the most challenging approach is represented by various sparse solvers (such as Least Angle Regression [9] used further), which identifies the most important basis functions from given set. However, sparse solvers are highly dependent on given ED, i.e. they need additional information in comparison to the previous methods.

2.2. Non-intrusive approach

Truncated PCE can be seen as a linear regression model with deterministic coefficients $\boldsymbol{\beta}$, which can be thus obtained by ordinary least square (OLS) regression. Estimated $\boldsymbol{\beta}$ thus minimize the sum of the squares of the differences between the results of original mathematical model $\mathcal{Y}$ corresponding to he input random vector $\boldsymbol{X}$ together called the experimental design (ED) and the results of surrogate model. Specifically, the vector of deterministic coefficients $\boldsymbol{\beta}$ is calculated using data matrix $\boldsymbol{\Psi}$ as

$$\boldsymbol{\beta} = (\boldsymbol{\Psi}^T\boldsymbol{\Psi})^{-1}\ \boldsymbol{\Psi}^T\mathcal{Y}. \quad (5)$$

The number of deterministic coefficients is directly connected to P, generally dependent on the number of input random variables M and the maximum total degree of polynomials p as can be seen in Eq. 4. Unfortunately, this leads to computationally highly demanding problems in case of large stochastic non-linear models. In order to reduce P, it is possible to select the best model represented by sparse set of basis functions. The best model selection is a broad scientific topic and several methods were proposed, here we use Least Angle Regression (LAR) [9, 8].

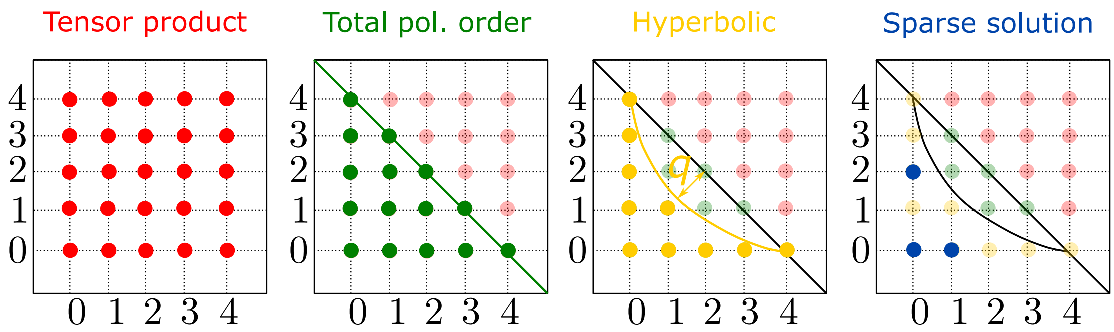

Fig. 1: Graphical comparison of basis sets constructed by selected methods for 2-dimensional examples with maximum polynomial order $p = 4$ for both input random variables.

Naturally, it is necessary to measure the approximation error of PCE . Error estimation in surrogate modeling of costly models is a specific and challenging task, since there is not enough samples for validation and one must usually use identical ED for training as well as for validation. Commonly used technique is the coefficient of determination R^2, which is well known from machine learning and statistics. However, this measure often leads to over-fitting and thus scientists are focused on more advanced techniques such as leave-one-out cross validation error Q^2. The estimated error is based on residuals between predictions of the surrogate model and the results of original mathematical model measured on ED, while excluding one realization in construction of surrogate model. The errors are calculated for all realizations in ED and further the average error is estimated. In case of PCE, it is possible to get Q^2 analytically from a single PCE based on all realizations in ED as follows [10]:

$$Q^2 = 1 - \frac{\frac{1}{n_{\mathrm{sim}}}\sum_{i=1}^{n_{\mathrm{sim}}}\left[\frac{g\left(\boldsymbol{x}^{(i)}\right)-g^{PCE}\left(\boldsymbol{x}^{(i)}\right)}{1-h_i}\right]^2}{\sigma^2_{Y,ED}}, \tag{6}$$

where $\sigma^2_{Y,ED}$ is a variance of experimental design obtained from results of the original mathematical model and h_i represents the ith diagonal term of the matrix $\mathbf{H} = \mathbf{\Psi}\left(\mathbf{\Psi}^T\mathbf{\Psi}\right)^{-1}\mathbf{\Psi}^T$.

2.3. Statistical Moments

The PCE is very efficient tool for post-processing allowing for analytical derivation of statistical moments of the QoI. The mean value is obtained from general formula of the first statistical moments as

$$\mu_Y = \left\langle Y^1 \right\rangle = \sum_{\boldsymbol{\alpha}\in\mathbb{N}^M} \beta_{\boldsymbol{\alpha}} \int \Psi_{\boldsymbol{\alpha}}(\boldsymbol{\xi})\, p_{\boldsymbol{\xi}}(\boldsymbol{\xi})\, \mathrm{d}\boldsymbol{\xi}. \tag{7}$$

Considering the orthonormality of the polynomials, the original integration is reduced to simple post-processing of the PCE deterministic coefficients. Namely, the mean value is equal to the first deterministic coefficient of the expansion

$$\mu_Y = \left\langle Y^1 \right\rangle = \beta_{\mathbf{0}}. \tag{8}$$

The second raw statistical moment, $\left\langle Y^2 \right\rangle$, is written as

$$\left\langle Y^2 \right\rangle = \sum_{\boldsymbol{\alpha}\in\mathcal{A}} \beta^2_{\boldsymbol{\alpha}} \int \Psi_{\boldsymbol{\alpha}}(\boldsymbol{\xi})^2 p_{\boldsymbol{\xi}}(\boldsymbol{\xi})\; \mathrm{d}\boldsymbol{\xi} = \sum_{\boldsymbol{\alpha}\in\mathcal{A}} \beta^2_{\boldsymbol{\alpha}} \left\langle \Psi_{\boldsymbol{\alpha}}, \Psi_{\boldsymbol{\alpha}} \right\rangle. \tag{9}$$

Similarly as in case of the mean value, it is possible to obtain the variance as the sum of all squared deterministic coefficients except the intercept (which represents the mean value), i.e.

$$\sigma^2_Y = \sum_{\substack{\boldsymbol{\alpha}\in\mathcal{A}\\ \boldsymbol{\alpha}\neq\mathbf{0}}} \beta^2_{\boldsymbol{\alpha}}. \tag{10}$$

The third and fourth statistical moments can be obtained similarly from deterministic coefficients for Hermite and Legendre polynomials [11], though its computation might be computationally expensive for large basis sets.

2.4. Sobol Indices

One of the most important tasks in UQ is the analysis of variance – the analysis of the influence of input variables on the variance of a mathematical model. Such information may be utilized to practically reduce the uncertainty of important input variables (material characteristics) used in mathematical model by experiments and measurements, which leads to a significant reduction in the uncertainty of the quantity of interest. Herein, the well-known ANOVA method represented by Sobol indices is employed. Unfortunately, it is still highly computationally demanding to evaluate Sobol indices via the classical double loop Monte Carlo method. However it was shown that there is a connection between PCE and the Hoeffding-Sobol decomposition [12] allowing for analytical derivation of Sobol indices.

PCE can be rewritten in the form of the Hoeffding-Sobol decomposition by a simple reordering of the terms:

$$g^{PCE}(\mathbf{x}) = \beta_0 + \sum_{\alpha \in A_{\mathbf{u}}} \beta_\alpha \Psi_\alpha(\xi), \tag{11}$$

where the set of basis multivariate polynomials dependent on selected input random variables $\mathbf{X_u}$ is

$$A_{\mathbf{u}} = \left\{ \alpha \in A^{M,p} : \alpha_k \neq 0 \leftrightarrow k \in \mathbf{u} \right\}. \tag{12}$$

Therefore, the first order Sobol indices can be analytically obtained directly from PCE as follows [12]:

$$S_i = \frac{\sum\limits_{\alpha \in A_i} \beta_\alpha^2}{\sigma_Y^2}, \tag{13}$$

where basis functions are selected as:

$$A_i = \left\{ \alpha \in A^{M,p} : \alpha_i > 0, \alpha_{j \neq i} = 0 \right\}. \tag{14}$$

Important information about the influence of input variables and all interactions can by expressed by total Sobol indices representing the first order influence and influence of all interactions, which can be obtained similarly from the following basis functions:

$$A_i^T = \left\{ \alpha \in A^{M,p} : \alpha_i > 0 \right\}. \tag{15}$$

2.5. Uncertainty Quantification with Python (UQPy)

The presented theoretical methods together with advanced iterative algorithms [13] were recently implemented to the sofware package UQPy [14]. UQPy represents multi-purpose complex software package for Python containing recently developed techniques for uncertainty quantification including PCE, more details can be found on official website, see QR codes in Fig. 2. UQPy contains several modules associated to common techniques for uncertainty quantification including surrogate models. PCE module in UQPy contains state-of-the-art techniques developed for advanced statistical sampling, efficient construction of the approximation (e.g. truncation schemes, sparse solvers) and its post-processing (e.g. Sobol indicies and complex statistical information derived from PCE). UQPy can be easily used for practical applications as well as for research, since it is an open-source package and anyone can contribute to the UQPy code once their contributions pass the quality checks.

3. Application: Concrete Bridge

PCE and UQPy is applied for statistical and sensitivity analysis the existing post-tensioned concrete bridge with three spans modeled as three simple spans. The super-structure of the mid-span is 19.98 m long with total width 16.60 m and it is crucial part of the bridge for assessment. In transverse direction, each span is constructed from 16 bridge girders KA-61 commonly used in the Czech Republic. Load is applied according to national annex of Eurocode for load-bearing capacity of road bridges by a special vehicle according to EN 1991-2.

Fig. 2: UQPy software: left) QR code leading to Git-Hub repository containing the open-source code in python, middle) the graphical logo representing the package, right) QR code leading to documentation of the package.

The NLFEM is created using software ATENA Science based on theory of non-linear fracture mechanics [15]. In order to reflect complex behavior of the bridge, the numerical model contains three construction phases as illustrated in Fig.3. The NLFEM consists of 13,000 elements of hexahedra type in the major part of the volume and triangular 'PRISM' elements in the part with complicated geometry. Reinforcement and tendons are represented by discrete 1D elements with geometry according to original documentation. The numerical model is further analysed in order to investigate three limit states: the ultimate limit state (ULS) (peak of a load-deflection diagram); first occurrence of cracks; decompression of prestressed concrete.

The stochastic model contains 4 random material parameters of a concrete C50/60: Young's modulus E ; compressive strength of concrete f_c; tensile strength of concrete f_{ct} and fracture energy G_f. Characteristic values of E, f_{ct}, G_f were determined from f_c according to formulas implemented in the fib Model Code 2010 [16] (G_f, E) and prEN 1992-1-1: 2021 (f_{ct}). The last random variable P represents prestressing losses with CoV according to JCSS: Probabilistic Model Code [17]. The stochastic model is summarized in Tab. 1. Mean values and coefficients of variation were obtained according to prEN 1992-1-1: 2021 (Annex A) for adjustment of partial factors for materials.

Tab. 1: Stochastic model of the numerical example.

Var.	Mean	CoV [%]	Distrib.	Units
f_c	56	16	Lognormal	[MPa]
f_{ct}	3.64	22	Lognormal	[MPa]
E	36	16	Lognormal	[GPa]
G_f	195	22	Lognormal	[Jm^2]
P	20	30	Normal	[%]

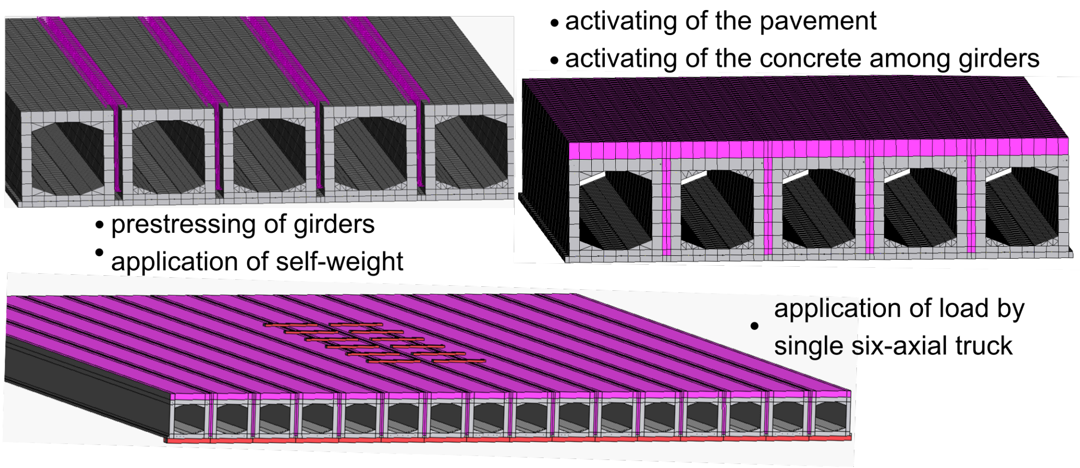

Fig. 3: Three construction phases of the bridge represented by NLFEM.

The experimental design (ED) contains 30 numerical simulations generated by Latin Hypercube Sampling (LHS). Note that each simulation takes approximately 24 hours and construction of the whole ED took approx. 1 week of computational time (PC with 12 cores and 64GB RAM). The PCE was created by UQPy and implemented iterative algorithm for selection of the best basis functions [13] and various maximum polynomial orders. From the estimated accuracy measured by Q^2, the maximum polynomial order $p = 4$ was selected for all three limit states. Once the PCE was created, it was possible to perform analytically statistical and sensitivity analysis.

Sensitivity analysis in form of Sobol indices was performed for all three limit states. The variability of decompression limit state is dominantly affected by P as expected (99%). The limit state of cracking already reflected influence of material parameters of concrete as can be seen in Fig. 4. The ultimate limit state was affected similarly as the second limit state – mainly by P, followed by G_f and less by E. Therefore it can be concluded that it is very important to perform additional survey of the existing structure in order to reduce uncertainty in P, since it is dominant variable in all limit states.

Statistical analysis was performed in sense of estimation of the first four statistical moments directly from PCE and also Monte Carlo simulation with 10^6 simulations used for construction of histograms of limit states. The first four statistical moments were further used for estimation of the analytical probability density function by Gram-Charlier expansion. Typical results can be seen in Fig. 5 corresponding to the decompression. It can be seen that analytical probability distribution function (PDF) corresponds very well to simulated data and thus it can be further used for estimation of design quantiles etc. Design values of resistance are typically very low fractiles and thus it is especially important to accurately approximate the cumulative distribution function (CDF) also in extreme values. As can be seen in Fig. 6, Gram-Charlier expansion (red) fits very well empirical CDF and it can be used for estimation fractiles which can not be accessed from existing ED (blue).

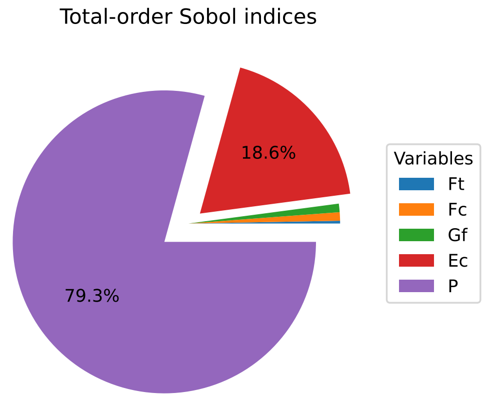

Fig. 4: Relative Sobol indices of the second limit state.

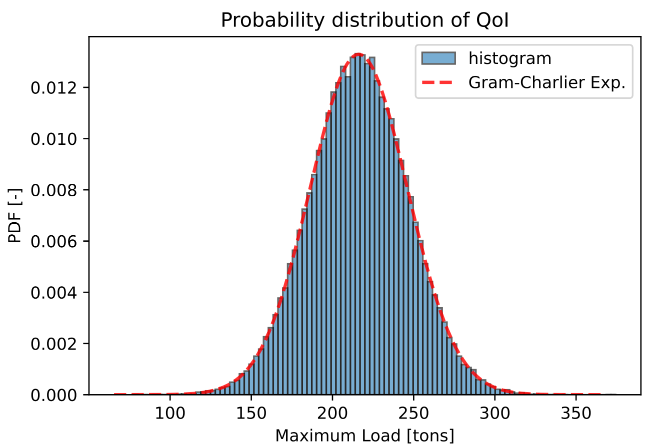

Fig. 5: Histogram of the first limit state: Decompression of the prestressed concrete.

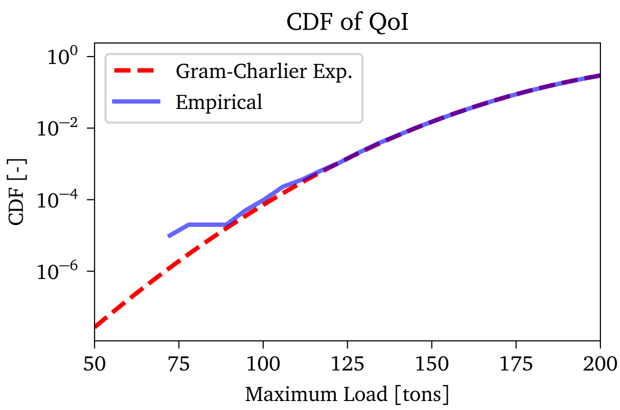

Fig. 6: CDF of the first limit state: Decompression of the prestressed concrete.

4. Discussion & Conclusions

The paper presented an application of PCE in statistical and sensitivity analysis of an existing structure – the post-tensioned concrete bridge with three spans represented by NLFEM. The whole process of UQ was done in the new version of open-source software package UQPy. It was shown that UQPy and PCE can be used for probabilistic analysis of real concrete structures. Note that it is clear from the obtained results of sensitivity analysis, the dominant influence on resistance has uncertainty of P. Therefore can be recommended to do additional survey of the bridge to reduce its uncertainty. Note that crossings by special vehicles require permits and thus surveys of bridges before such crossings should be prescribed in order to reduce the uncertainty in P. Moreover obtained results support a reduction of the stochastic model (and thus dimensionality) of the practical example leading to higher computational efficiency of UQ. Further work will be also focused on reliability assessment of the bridge using obtained results from statistical and sensitivity analysis. Naturally, it will be necessary to extend the stochastic model of the example in order to cover additional aspects affecting reliability of the bridge. Specifically, reliability of post-tensioned concrete bridges is commonly dominated by corrosion of prestressing tendons [18, 19]. Although a recent survey of the bridge did not present any existing corrosion, it could be included as additional uncertain parameter for further studies. Moreover, additional uncertainties in traffic loading can be also important for further reliability analysis [20].

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